Multi-Cycle Optimal Taxi Routing with E-hailing

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ABSTRACT

In this study, an optimal taxi routing problem is investigated for a single taxi that accounts for multiple cycles of pick-up and drop-off into the future to improve the utilization of taxis. An optimization-based approach is developed that aims at maximizing the total expected payoff for a single taxi over multiple cycles. The method is applicable in large-scale networks and solutions are obtained in acceptable computational time. To assess the merits of the methods, a simulation is carried out that mimics a single taxi starting from various locations for a two-cycle operation according to the optimal policy in the city of Shanghai. The simulation results indicate that our methods substantially improve the unit profit and occupancy rate of taxis starting from most nodes over two cycles.

1 Introduction

1.1 Motivation

Taxis play an important role in providing on-demand mobility in the urban transportation system. Compared to other forms of public transportation, the advantages of taxis include speediness, privacy, comfort, door-to-door service and 24/7 operations. Traditionally, vacant taxis cruise on roads searching for customers. In recent years, thanks to the advance of smartphone technology, e-hailing applications (e.g., Uber, Lyft, and Didi) are widely adopted by the drivers to receive requests from nearby customers. An occupied taxi usually takes a direct route to the customer's destination, not unlike regular commuters. However, there is no guarantee that the driver can find a new customer after dropping off the previous customer at the destination. Vacant taxis cruising on roads not only result in wasted gas and time for taxi drivers but also generate additional traffic in a city. Therefore, how to improve the utilization of taxis is of importance to both taxi drivers and the society.

In an earlier study by the co-authors Hu et al. (2012), a dynamic programming model of routing vacant taxis was proposed to depict the decisions at intersections according to the passenger arrival rate. However, the expected search time is only minimized for the next customer, which might be inefficient in the long run. For example, driving to the airport might not minimize the search time for the next customer, but it brings in a higher chance of a long trip for the next customer and thus the profit is higher in a long period. For this reason, experienced taxi drivers would not simply make their customer-search decisions depending on the current searching time/profit, but would also consider the subsequent possible states that could be encountered. In this study, an optimal taxi routing problem is investigated for a single taxi that accounts for multiple cycles of pick-up and drop-off into the future.

1.2 Literature Overview

Since the early 1970's, a large number of studies on taxis have been conducted. See Salanova et al. (2011) for a recent review. Static or myopic taxi routing modeling and optimization is beyond the scope of this study.

For modeling multi-period taxi services, Yang et al. (2005) presented a spatially aggregated taxi service equilibrium model with endogenous service intensity. However its main purpose is to understand the demand-supply interaction instead of improve taxi utilization. Wong et al. (2015) developed a sequential logit-based vacant taxi behavior model predicting searching paths as a sequence of choices of adjacent zones. The destination choice is restricted to adjacent zones, while an experienced driver chooses destinations over the whole network (e.g, going back to airports and major business districts after dropping off customers in a remote residential area). Furthermore, the search behavior model is based on zones and cells instead of the road network.

Models and algorithms developed for non-myopic vehicle routing problem (VRP) under uncertainty with look-ahead policies and rolling horizons (Mitrović-Minić et al., 2004; Thomas and White, 2004; Ferrucci et al., 2013)e.g., might provide insights for taxi routing problems in terms of accounting for future unknown demand and efficient solution algorithms. It is however recognized that the taxi problem is different. In a typical VRP, the service of a customer does not bring the vehicle to another location, while a taxi does and the destination is not known until the request is taken. This significantly increases the geographic spread of taxi movements. In addition, a taxi (without carpooling service) can serve only one quest at one time and a new request does not come up until the old request is finished (unless a dispatcher is sending request during the previous ride).

Accounting for future states in taxi searching behavior requires sound models of geographic and temporal distributions of taxi demand. Several methods have been proposed to predict taxi demand distribution (Yuan et al., 2011; Moreira-Matias et al., 2012; Qian and Ukkusuri, 2015; Hwang et al., 2015), which could be combined with the optimal taxi routing model.

1.3 Contributions

Define a trip cycle consisting of the vacant taxi trip from the destination of the previous occupied trip to the pick-up place for the next customer, and the subsequent occupied trip from pick-up place for transporting the customer to his/her destination. A non-myopic, multi-cycle taxi routing optimization problem is studied with the following intended contributions:

- The optimization problem takes into account future states by considering both the intensity of future customer demand and their destinations over multiple cycles.

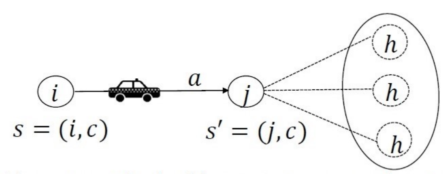
- Instead of zone/cell-based movements, the routing decisions are based on the physical road network, which enables potentially better understanding of taxi drivers' behaviors as well as more practical recommendations for taxi drivers.
- Practical implementations are proposed to solve the multi-cycle optimal taxi routing problem in reasonable time. The solution is compared with observed searching behaviors from GPS trajectories in a mega city to demonstrate the advantage of the multi-cycle approach.

The remainder of the paper is organized as follows. Next section introduces mathematical notation and preliminaries of taxi movement, followed by formulations of a multi-cycle taxi routing problem. Section 3 presents the numerical experiments on evaluating the benefits of our approach. Finally, Section 4 concludes the paper and discusses future work.

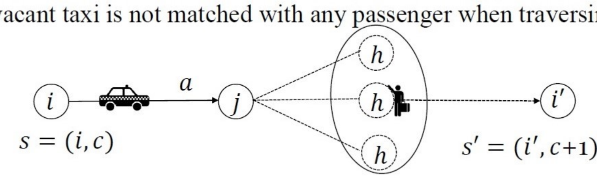
2 Formulation of the Multi-Cycle Optimal Taxi Routing Problem

2.1 Taxi Movements in a Network

A taxi travels in a traffic network $G = (N, A)$. N is the set of nodes and A the set of links. There is at most one directional link, a, from the source node i to sink node j. τ_a is the travel time on link a. For simplicity, link travel times are assumed time-independent and deterministic, but the formulation can be generalized to account for time-dependent and/or stochastic link travel times. $A(i)$ is the set of downstream links of i, and $B(j)$ the set of upstream links of j. The taxi is actively searching for, or carrying passengers during a planning horizon with C pick-up and drop-off cycles.



a) A vacant taxi is not matched with any passenger when traversing link a



b) A vacant taxi is matched with a passenger when traversing link a

Fig. 1 : Illustration of the potential passenger matching process on a link

The routing problem is meaningful only when a taxi is for hire. The state of a taxi, s, is described by node i and current cycle $c \in \{1, 2, \ldots, C\}$. The action set for state s is the set of outgoing links $A(i)$. For a given state s and action $j \in A(i)$, two types of transition to a new state s' could happen (see Fig. 1). a) The taxi is not matched with any passenger when traversing link $a = (i, j)$, and s' is associated with sink node, j. The taxi remains in the same cycle c. b) The taxi is matched with a passenger when traversing link a, and s' is associated with the destination node of the passenger i' . The taxi enters the next cycle $c + 1$ once dropping off the passenger. By definition, any state associated with cycle $C + 1$ is a terminal state. To calculate state transition probabilities, the passenger matching probability on a link (Section 2.2) and passenger destination probabilities (Section 2.3) are needed.

2.2 Passenger Matching Probability on a Link

Passengers arrive at link a following a one-dimensional space-time Poisson process with rate λ_a per hour per mile. For modeling convenience, these are simplified as time Poisson processes at nodes, and the arrival rate at node j (per hour), $\lambda_j = \sum_{a \in B(j)} \lambda_a l_a$, where l_a is the length of link a. Arrivals at different nodes are independent. The combined process over all nodes is also a Poisson process with arrival rate $\lambda = \sum_{j \in N} \lambda_j$. In practice, demand rate λ_j is often approximated by observed met demand rate. Statistical analysis can be carried out to build a predictive model for the demand rate as a function of built environment variables (e.g., residential density, and employment by business type such as hotel and nightclub), time of day, and weather condition (Phithakkitnukoon et al., 2010; Moreira-Matias et al., 2012).

When e-hailing is used, the nearest vacant taxi to a passenger gets matched to the passenger. Vacant taxis around node n at any given point of time follow a two-dimensional spatial Poisson distribution with density γ_n. For a given node n, the probability of a vacant taxi r miles away (based on right-angle travel) being the nearest vacant taxi is the probability of no vacant taxi in a square rotated at 45 degree centered at node n with area equal to $2r^2$, namely,

$$P_n(r) = \exp(-2\gamma_n r^2). \tag{1}$$

The set of potential pickup nodes is limited to those that are within a certain distance to the vacant taxi. Let $N(j)$ denotes the sets of nodes within a certain matching distance R to node j. That is, R is the farthest distance between a passenger and a taxi where a request can go through. The combined process over all nodes in $N(j)$ is also a Poisson process with arrival rate $\lambda_{N(j)} = \sum_{j \in N} \lambda_j$.

Consider a vacant taxi with e-hailing traversing link a. It gets matched with a passenger at node h when the following conditions are all satisfied.

- A passenger arrives at node h during the traversal time τ_a,
- The arrival at node h is earlier than arrivals on all other nodes in $N(j)$,
- The taxi in question is the nearest vacant taxi to node h.

The probability of having at least one arrival from any node during τ_a is $1 - \exp(-\lambda_{N(j)}\tau_a)$. The probability that an arrival from node h is earlier than all other nodes is $\frac{\lambda_h}{\lambda_{N(j)}}$. The product of the two probabilities is the probability that the earliest arrival during τ_a happens at node h. The matching probability, $p_{a,h}$, is the product of the probability that the earliest arrival during τ_a happens at node h and the probability that the taxi in question is the nearest vacant taxi to node h, namely,

$$p_{a,h} = \begin{cases} \dfrac{\lambda_h}{\lambda_{N(j)}} \left(1 - \exp(-\lambda_{N(j)}\tau_a)\right) \exp(-2\gamma_h \mathcal{L}^2_{h \to a}), & \text{if } h \in N(j) \\ 0, & \text{otherwise} \end{cases}, \tag{2}$$

where $\mathcal{L}_{h \to j}$ is the right-angle distance from node h to link a, which can be approximated as the distance to the middle point of link a.

For those taxis that pick up passengers along the roads without e-hailing, it usually requires that the taxi and passenger to be no more than 1 block away from each other, thus the pickup nodes without e-hailing can be restricted to a subset of the pickup nodes with e-hailing.

2.3 Passenger Destination Probabilities

The probability of a passenger picked up at node h having node k as the destination, $p_{h \to k}$, can be approximated by the observed fraction of passengers picked up at node h going to k. When no passenger pick-up is observed at node h, the probability is undefined. To resolve this problem, the study area is divided into zones such that any zone has strictly positive number of pick-ups. Let node h be in zone $\mathcal{H}$ and node k in zone $\mathcal{K}$. Assume each node in zone $\mathcal{K}$ has equal probability of being the destination node, and the destination probability is

$$p_{h \to k} = \begin{cases} \dfrac{p_{\mathcal{H} \to \mathcal{K}}}{m_{\mathcal{K}}}, & \forall \mathcal{H} \neq \mathcal{K} \\ \dfrac{p_{\mathcal{H} \to \mathcal{K}}}{m_{\mathcal{K}} - 1}, & \forall \mathcal{H} = \mathcal{K}, \forall h \neq k \\ 0, & if h = k \end{cases}, \tag{3}$$

where $p_{\mathcal{H} \to \mathcal{K}}$ is the probability of a passenger picked up in zone $\mathcal{H}$ having zone $\mathcal{K}$ as the destination zone, and $m_{\mathcal{K}}$ is the number of nodes in zone $\mathcal{K}$. The equal probability assumption can be easily relaxed.

The sizes of the zones should be designed carefully based on the required level of modeling accuracy and the information available to the modeler. An unnecessarily large zone would mask traffic pattern differences that might be important for taxis finding customers. If the sizes were too small, the relevant data collected would be statistically unreliable, and the number of samples in each zone would be insufficient to provide representative means on the model parameters. In practice, it is typical to use the traffic analysis zones (TAZs) as the basis for calculating passenger destination probabilities.

2.4 State Transition Probabilities and the Optimization Problem

Let $s = (i, c)$, where $i \in N$ is the current node and $c \in \{1, 2, ..., C\}$ is the current trip cycle. For a given state $s = (i, c)$ and action $a \in A(i)$ with a sink node j, the transition probability, $p_{ss'|a}$ with $s' = (i', c')$, is defined as follows:

$$p_{ss'|a} = \begin{cases} 1 - \sum_{h \in N(j)} p_{a,h}, & \text{if } i' = j, c' = c, c \leq C \\ \sum_{h \in N(j)} p_{a,h}\, p_{h \to i'}, & \text{if } c' = c + 1, c \leq C, \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

In the first case, no passenger is matched along link a, and the taxi arrives at the sink node j, staying in the same cycle $c' = c$. In the second case, a passenger from node h with destination i' is matched, and the taxi arrives at node i' after picking up the passenger from node h and carrying the passenger from h to i', both following shortest paths. The probabilities are summed over all possible h. The taxi is in the next cycle $c' = c + 1$ after dropping off the passenger.

It follows that the immediate profit of going from state s to s' given action a can be written as follows:

$$g_{ss'|a} = \begin{cases} -\alpha\tau_a, & \text{if } i' = j, c' = c, c \leq C \\ \dfrac{\sum_{h \in N(j)} \left[-\alpha(\tau_a + \mathcal{T}_{j \to h} + \mathcal{T}_{h \to i'}) + F(d_{h \to i'})\right] p_{a,h} p_{h \to i'}}{\sum_{h \in N(j)} p_{a,h}\, p_{h \to i'}}, & \text{if } c' = c + 1, c \leq C, \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where α is the taxi operating cost per unit time, and $\mathcal{T}_{j \to h}$ ($\mathcal{T}_{h \to i'}$) is the shortest path travel time from node j to h (h to i'). $F(d_{h \to i'}) = f_0 + \beta max(0, d_{h \to i'} - d_0)$ is the taxi fare of an occupied trip from pickup node h to destination node i', where $d_{h \to i'}$ is the occupied travel distance from node h to i'. That is, the taxi fare is f_0 for the first d_0 kilometers and an additional charge of β for every succeeding kilometer, a fare structure adopted in Shanghai*.

Let $V^*(s)$ denotes the optimal expected payoff starting from state s. The taxi driver chooses the action at each state s to maximize the expected payoff that is the sum of the expected immediate payoff and the expected downstream payoff, which is the expectation of the payoff over all possible next state s'. The optimal expected payoff is obtained by solving the Bellman equation(Bellman, 1957) as follows:

$$V^*(s) = \begin{cases} \max\limits_{a \in A(s)} \sum\limits_{s'} \left[g_{ss'|a} + V^*(s')\right] p_{ss'|a}, & \forall s | c \leq C \\ V^0(s), & \forall s | c = C + 1, \end{cases} \tag{6}$$

$V^0(s)$ is the value function at any terminal state, and can be set as estimated average payoff from the end of the multi-cycle planning horizon to the end of the driver's working hours.

The optimal routing policy is then written as follows:

$$\mu^*(s) = \begin{cases} \arg max_{a \in A(s)} \sum\limits_{s'} \left[g_{ss'|a} + V^*(s')\right] p_{ss'|a}, & \forall s | c \leq C \\ i, & \forall s | c = C + 1. \end{cases} \tag{7}$$

3 Numerical Analysis

3.1 Data and Network

We use historical GPS dataset generated by urban taxis in Shanghai, China, in April, 2015. This dataset contains detailed trajectories of each of the 13,657 active taxis at 10-sec intervals, with a status indicator to identify whether

*source: https://www.travelchinaguide.com/cityguides/shanghai/transportation/taxi.htm a taxi is occupied or not. The sample of these urban taxis represent approximately 25% of the entire Shanghai urban taxi population, which offer an adequate sample size to deduce the movement of Shanghai urban taxis.

All data were pre-processed to remove erroneous/useless records, such as coordinates located outside the city boundary. It is noted that some of the extracted occupied and vacant trips were actually non-existence, such as the trips with exceptionally short travel distances during a long period or super short trip travel times (i.e., occupied trips shorter than 1 mile or 1 min). The recording of these extracted trips could be due to the GPS device malfunctions, poor connectivity to satellites in the urban areas surrounded by high-raised buildings, or human error by taxi drivers who operated the devices. Therefore, a data screening process was carried out to eliminate such trips.

Shanghai roadway network is comprised of 13,531 nodes and 30,167 links (directed/not including connectors) with dense-distributed roadway network in urban area and relatively sparse roadway network in suburban area. The travel time on each link of the network is computed based on the speed limit by road type. Congested travel time could potentially result in better solution, but the difference is not expected to be large. Shortest paths and travel times between all nodes are then pre-calculated and stored in a lookup table.

TAZs are used as the basis for calculating passenger destination probabilities. The study area, including 16 districts of Shanghai, is divided into 4,518 zones. Each zone has a certain degree of homogeneity of land use, and thus entails the assumption that each node within a destination zone has equal probability of being the destination node.

3.2 Experiment Settings

In particular, we extract taxi trajectories covering two pickup and drop-off cycles, starting at morning peak (7:30-9:30 a.m.) of a representative weekday in April, 2015. Passenger arrival rates and empty taxi density rates are assumed time-invariant during the studied two cycles, which can be derived from the historical data.

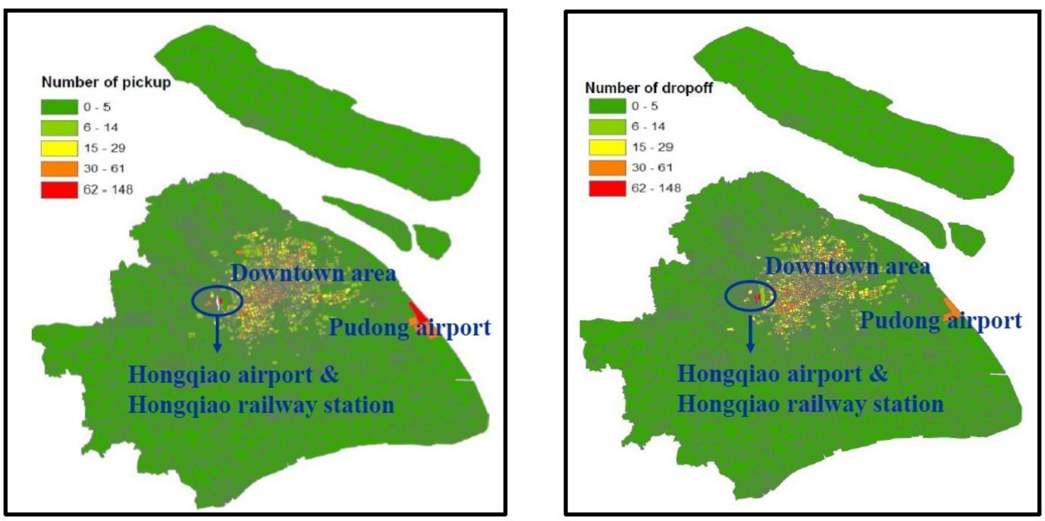


Fig. 2 : Spatial distribution of pickup and drop-off across TAZs

Fig.2 illustrates the number of pickup and drops in each TAZ in Shanghai from the extracted data. The yellow/orange/red zones represent more trips involved in those zones. Most of the urban taxi drivers operated within city centers, and to-and-from Pudong airport, which is located far way from the central business areas and residences. It is also noted that most short trips concentrate in downtown area, where most business, social, and cultural activities converged day and night, while long trips spread broadly and their hotspots center on airports and railway stations.

The set of pickup nodes $N(j)$ was the set of nodes centered at node j within a given distance R of 2 kilometers. In the day time, the taxi price is RMB 14 for the first 3 kilometers and an additional charge of RMB 2.5 for every succeeding kilometer, i.e., the taxi fare $F(d_{h \to i'}) = 14 + 2.5max(0, d_{h \to i'} - 3)$.[†] As for the operating cost, α was set to be 0.8 RMB/min.

The optimization algorithms are coded in Python 3.5. All computations are carried out on a workstation with eight-core 3.0GHz Xeon E5-1660 processors and 64GB RAM. The multi-cycle optimal taxi routing problem using

[†]source: https://www.travelchinaguide.com/cityguides/shanghai/transportation/taxi.htm

Shanghai roadway network was solved within 7 h. This is a reasonable time considering the large network size and the huge state space. Since the reported run-time for the optimization is based on sequential processing of different starting nodes, the actual run-time when starting nodes are run in parallel will be lower.

To assess the performance of the proposed approaches on metrics that are based on elapsed time, e.g. unit profit, occupancy rate, we conduct a simulation that mimics a single taxi starting from various locations for a two-cycle operation according to the optimal policy in the city of Shanghai.

The optimal policy $\mu^*(i,c)$ specifies an action $a=(i,j)$, i.e., $\mu^*(i,c) \to a, a \in A(i), c \leqslant C$, for each state $s=(i,c)$. The execution of the optimal policy from any given node i could result in multiple realizations of trajectories due to the random processes of passenger arrival, competition with other vacant taxis, and passenger destinations. To sample a trajectory, first the location of the matched passenger is sampled according to matching probability (Eq. 2). If no matching happens(with probability $1-\sum_{h\in N(j)} p_{a,h}$), the taxi moves to node j and remains in the same cycle. The sampling of location of matched passengers is then repeated. If matching location h is sampled (for taxi moving from i to j), then the passenger arrival happens during the traversal of link a and the elapsed time until picking up at node h is $\tau_a + \mathcal{T}_{j\to h}$. Next the passenger destination i' is sampled according to Eq.(3), and the elapsed time until passenger drop off is $\tau_a + \mathcal{T}_{j\to h} + \mathcal{T}_{h\to i'}$. At i', the cycle index is advanced by 1, and the sampling continues. The sampling stops at a terminal state ($c = C+1$).

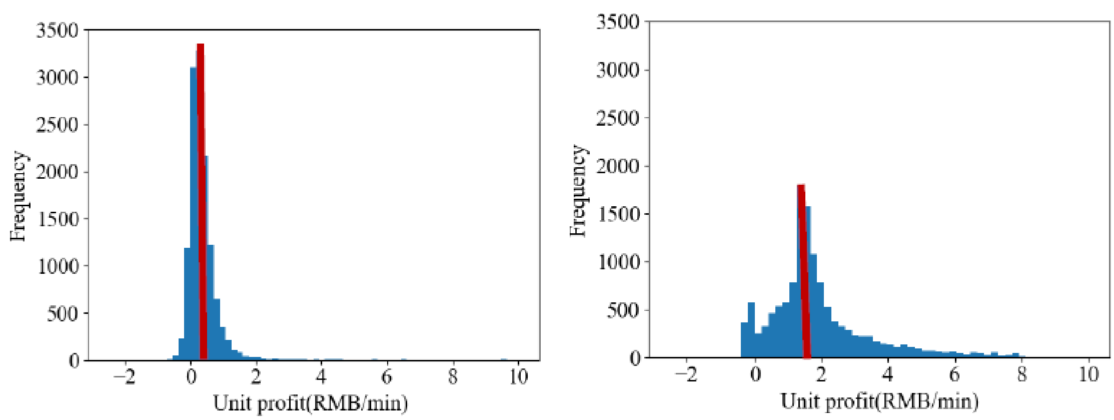
Taxi trajectories over two cycles based on the optimal policy are simulated by drawing the same number of taxis starting from each node in the observed dataset, and repeat this routine over all nodes in the network. Thus the size of the simulation sample is equal to the size of the original observed dataset.

3.3 Results

We employ the following metrics over two pick-up and drop-off cycles to compare the performance of our approach against real observations:

(1) Unit profit (RMB/min)

(2) Occupancy rate: the quotient between occupied time and the total working time



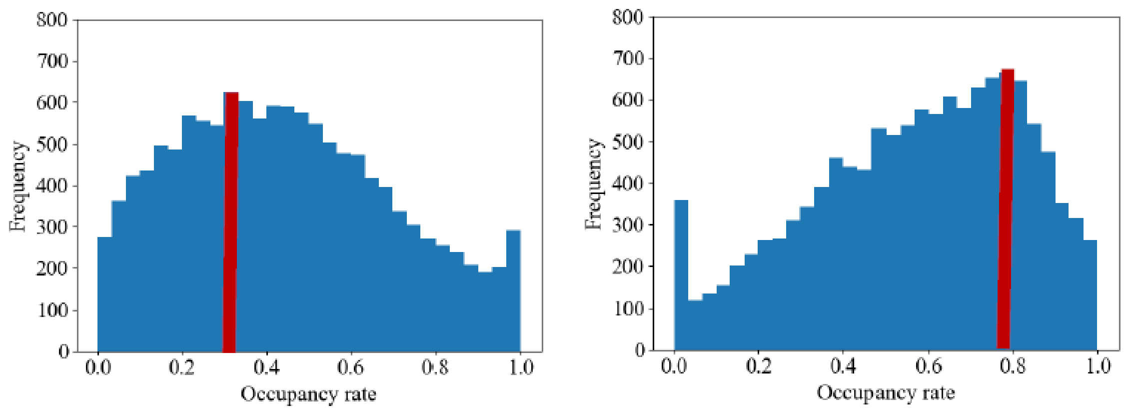
(a) Observed data

(b) Simulated data from optimal policy

Fig. 3 : Distribution of unit profit(RMB/min)

We first present a comprehensive comparison over all nodes in the network. Fig.3 shows the distribution of unit profit (RMB/min) from real observations and simulated data from optimal policy over two cycles. The simulated data from optimal policy peaks at approximately 1.33, a larger value compared to the real observed data, which peaks at 0.67. We define it as a Success at node i if the average unit profit of the simulated data is larger than that of the observed data at this node. We observe that the simulated data from optimal policy is able to achieve 89.56% Success rate over all observed starting nodes.

Fig.4 shows the distribution of occupancy rate of observed data and simulated data from optimal policy. In general, high occupancy rate is achieved from optimal policy, and it peaks at approximately 0.76, larger than the observed data, which peaks at 0.33. Similarly, the Success rate for occupancy rate is 88.94%. While it is not the optimization criterion for our approach, we are still able to perform better than real observations with respect to unit



(a) Observed data

(b) Simulated data from optimal policy

Fig. 4 : Distribution of occupancy rate

profit and occupancy rate for taxis starting from most nodes. To understand this, note that "less time spent searching passenger for" coincides with "more time spent making money". Thus, there is more profits possible if a taxi operates at high occupancy rate.

Next we consider trajectory details based on the optimal policy for taxis staring from some typical locations.

Consider a taxi starting from a typical residence, the optimization-based approach does not always generate route that guarantees a higher Unit profit/Occupancy rate than the observed one, however, the expected unit profit from optimization (1.03 RMB/min) is almost twice the observed expected unit profit (0.54 RMB/min). Here our claim is that our approach has high potential to guide taxis to achieve larger unit profit/occupancy rate for taxis starting from this particular location.

For taxis starting from Pudong airport, the best policy is to move around until it gets matched with a passenger according to our model, which is consistent with real observations. In real life, it is observed that taxis at Pudong airport usually wait in a queue in the airport taxi stand to pick up potential passengers. Since Pudong airport is located far away from central business and residential areas, it seems more worthwhile for taxis to wait in line to pick up passenger in the airport than to going back to some taxi hot spots. It would not be too difficult to extend this research to accommodate waiting behavior of empty taxis, which are ignored in this initial study.

4 Conclusion

In this study, we investigate an optimal taxi routing problem for a single taxi that accounts for multiple cycles of pick-up and drop-off into the future to improve the utilization of taxis. We develop optimization-based approaches that aim at maximizing the total expected payoff for a single taxi over multiple cycles. The multi-cycle approach is applicable in large-scale networks and solutions are obtained in an acceptably computational time. To assess the merits of our methods, we conduct a simulation that mimics a single taxi starting from various locations for a two-cycle operation according to the optimal policy over the city of Shanghai. The simulation results indicate that our method substantially improve the unit profit and occupancy rate of taxis starting from most nodes over two cycles.

In this initial study, the multi-cycle optimal taxi routing approach assumes time-invariant arrival rates and empty taxi density during the studied two cycles; more realistic situations should take into account time-dependent customer demand and empty taxi distribution in the future. Formulating the problem as a Markov decision process in a time-space network problem is both computationally intractable (the problem is huge and was beyond the capabilities of classical math programs) and challenging(if we want to consider real-time impact within the multi-cycle framework). Approximate Dynamic Programming (ADP) provides a potential tool for calculating the future impact of a current decision (see, e.g., Larsen et al., 2004; George and Powell, 2006) . ADP approximates the value function based on some methods (e.g. reinforcement learning, Q-learning and simulation), and ultimately avoids the evaluation of all possible states.

Note that since we consider single-taxi routing optimization only, it is likely that many taxis are guided to the same area. As a result, there are significantly more empty taxis available than demand in this area. Future research should consider dynamically rebalancing the empty taxis by moving only part of the nearby taxis.

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