Cooperative Scheme - An Alternative Approach to an Equitable and Pareto-Improving Transportation System

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ABSTRACT

We propose an alternative approach to an equitable and Pareto-improving transportation system based on cooperation among travelers assisted by defector penalty. In the theoretical analysis, it is shown that when the value of time (VOT) is bounded from above, a Pareto-improving cooperative scheme without financial transactions always exists, in which case the defector penalty is high enough so that all travelers cooperate. A more practical and potentially more appealing case is discussed where a certain number of defectors exist while the travel time of cooperators is strictly better than that in UE. A mathematical programming problem is formulated for the optimal cooperative scheme problem in a general network with Pareto-improving constraints and practical considerations on the length the cooperation cycle. Computational tests on a simple network and solutions are evaluated in terms of efficiency improvement (total system travel time) and equitability (Gini index).
INTRODUCTION AND LITERATURE OVERVIEW

User Equilibrium (UE) is based on the assumption that travelers behave selfishly in a non-cooperative manner to minimize their own travel cost. System Optimum (SO) is a traffic state where the total cost of the system is minimized. However, SO is not stable as drivers on slow routes will likely shift to the fast routes, and cause the system to revert back to UE. Therefore, drivers need to be penalized through charges or compensated through rewards for the system to move towards SO. The study of congestion pricing is traced back to the early twentieth century when (1) recommended a tax to be levied on any market activity that generates negative externalities. (2) proposed a time-varying toll that could completely eliminate queuing delay, and thereby maximize system efficiency. A plethora of studies have been conducted in this area since then.

Despite its theoretical appeal, congestion pricing continues to be a hard sell to people. Major proposals have been remonstrated by public or political opposition. For example, cordon tolling schemes for Edinburgh and Manchester in the UK were rejected by public referenda (2005 and 2008). An online petition to the UK government (2007) attracted more than 1.8 million signatures against road pricing, and effectively put an end to plans for a national scheme in the UK for the time being. A cordon toll plan for New York City was stopped by the New York state legislature (2008) when it declined to vote on the proposal. These setbacks illustrate the difficulties of designing congestion pricing schemes that are both efficient and publicly acceptable (3).

There are a wide range of factors for the setbacks, and equity is one of the most cited. Congestion pricing sometimes is characterized as a “regressive tax” (4) in that high income travelers who usually have a high value of time (VOT) could benefit at the cost of low income travelers’ loss. Innovative solutions to the equity issue have focused on the so-called Pareto-improving schemes, where no traveler is worse off compared to the no-toll case. Examples include a hybrid scheme between rationing and pricing (5, 6), alternating charging a given fraction of the drivers (7), Pareto-improving pricing (8), credit-based scheme (9), tradable credit scheme (10–13), and toll-and-subsidy scheme (14–16).

The cooperative scheme (CS) proposed in this research is an extension of the hybrid scheme between rationing and pricing, first proposed by (5) for a single bottleneck with flexible demand, and later adopted and extended by other researchers, e.g., (6). The major distinction is that the CS in this research is applied to route choice in a general network, where the ”rationing” is origin-destination (OD) and route-specific. This flexibility allows for potentially more room for improvement in both efficiency and equitability, but also renders a more challenging problem.

It is hypothesized that traveler cooperation will bring about transformative changes to how the transportation system is managed (17), and its implementation can be accelerated by technologies including connected and autonomous vehicles (CAV). Specifically, with fully autonomous vehicles, the barrier to participation in the cooperative scheme due to cognitive constraint (e.g., inertia against regular switching to potentially unfamiliar routes/departure time) and disruption to the execution due to human errors (e.g., failing to follow specified route) can be significantly reduced, and even eliminated.

As reviewed in (17), cooperation has been studied extensively in behavioral economics and game theory to resolve social dilemmas such as Prison’s Dilemma (18). Evolutionary game theory provides a competent theoretical framework for addressing the subtleties of cooperation in such situations (19–22). (23) conducted experiments on humans playing two-person route choice games in a computer laboratory to study decision behavior in repeated games. Results show that a taking-turn strategy that achieves SO emerges after the two players have enough experience to perceive the value of cooperation. However, computer simulations and additional experiments indicate that oscillatory cooperation in route choice games with four players emerge only after a long time period (rarely within 300 iterations).

Almost all traffic equilibrium studies make the assumption that travelers are non-cooperative, for a good reason. With the large number of travelers, the time it takes for cooperation to emerge is too long.
for the assumption to be practically valid. Penalty to defectors (people who do not cooperate) has been suggested (17, 23) to promote cooperation. This study operationalizes the idea in both theoretical and computational analyses.

The remainder of the paper is organized as follows. The next section presents a theoretical analysis of the CS for both homogenous and heterogenous VOT. A mathematical formulation of the optimal CS problem is then presented along with computational tests. Finally conclusions and future directions are provided.

THEORETICAL ANALYSIS

Consider a single-OD network (Figure 1) with fixed demand $d$, connected by two routes: Fast Route with a travel time $t_F$ at SO and Slow Route with a travel time $t_S$ at SO. The two routes have the same travel time at UE, $t_{UE}^F$, and $t_F < t_{UE}^F < t_S$. $x_F$ and $x_S$ are the SO flows (positive integers) on Fast Route and Slow Route respectively, and $x_F + x_S = d$. Travel time is a strictly increasing function of flow.

A CS is defined as travelers taking turns to use Fast Route. Participants of the cooperative scheme are called “cooperators”. They use Fast Route on some days and Slow Route on other days following the guidance of a central controller to maintain an SO flow pattern on each day, even though the composition of the flow varies from day to day due to turn taking.

When all travelers are cooperators, a naive turn-taking strategy is such that in each cycle of $d$ days, each cooperator uses Fast Route for $x_F$ days and Slow Route for $x_S$ days. The average travel time for each cooperator over a cycle is the average SO travel time:

$$t_{SO} = \frac{x_F t_F + x_S t_S}{d}$$

(1)

It is evident that $t_F < t_{SO} < t_{UE}^F < t_S$.

Such a scheme can be directly extended to multiple ODs and more than two routes per OD when all travelers are cooperators. For any OD $(m, n)$ with demand $d_{mn}$, in a cycle of $d_{mn}$ days, each cooperator uses the routes in proportion to the SO route flows (positive integers). Since SO flows are maintained for each OD, they must be maintained for the network, although the composition of the flows varies. In classical traffic assignment problems, flows are fractional numbers instead of integers, however, when the flow is large enough, the difference of rounding to integers is negligible (not to mention that flows are integers in reality).
A shorter cycle is preferred as it demonstrates the value of turn-taking in shorter time and thus more appealing for getting public acceptance. With an improved strategy, cooperators take turns by blocks. Let $g$ be the greatest common factor of $x_F$ and $x_S$. Cooperators are grouped into $d/g$ blocks. In each cycle of $d/g$ days, each block of cooperator uses Fast Route for $x_F/g$ days and Slow Route for $x_S/g$ days. The resulting average travel time over a cycle is still $t^{SO}$. In practical applications, the cycle length might need to be controlled below a fairly small number, say, 5 working days, to have a realistic chance of acceptance.

In such cases, indifference to small travel time differences (e.g., a 5-minute difference for a 1-hour trip) can be exploited such that a small number of approximately equal-sized blocks result in approximately equal average travel time for each cooperator with the differences under a certain threshold. A turn taking strategy of a small number of blocks of cooperators is demonstrated in Figure 2. On day 1, blocks 1 and 2 use the fast route. On day 2, block 2 switches to slow route and block 3 switches to fast route. And on day 3, block 1 and 2 interchanges their route. Thus each block of cooperator uses fast route for 2 days and slow route for 1 day.

![Figure 2: Turn-taking of 3 blocks of cooperators over a cycle of 3 days](image)

The cooperative scheme is not stable, as a “defector” who stays on Fast Route all the time has a lower travel time of $t_F$ than a cooperator. As more travelers defect, the cooperative scheme is broken and the system reverts back to UE. One way to maintain the cooperative scheme is to impose a defector penalty $\tau$ to make defection more costly than or at least as costly as cooperation.

**Homogeneous VOT**

If a single VOT of $\bar{\beta}$ is assumed for all the travelers, $\tau \geq \bar{\beta}(t^{SO} - t_F) = \bar{\beta}(t_S - t_F)x_S/d$.

When $\tau > \bar{\beta}(t_S - t_F)x_S/d$, defection is more costly than cooperation, and thus no defectors exist and no financial transactions.

When the penalty is exactly $\bar{\beta}(t_S - t_F)x_S/d$ and the penalty collected from defectors is distributed evenly to cooperators, defection is as costly as cooperation. Multiple cooperative schemes exist with different number of defectors, $n$, ranging from 0 to $x_F$. When $n = 0$, the cooperative scheme is the same as described above. When $0 < n < x_F$, cooperators have to use Fast Route proportionally less often than when $n = 0$ to maintain the SO flow pattern, and their average travel time

\[
t^{CS}(n) = \frac{(x_F - n)t_F + x_S t_S}{d - n}. \tag{2}
\]

$t^{CS}(n)$ is an increasing function of $n$, and $t^{CS}(n) > t^{CS}(0) = t^{SO}, \forall n > 0$. In other words, given that the total system travel time remains at the SO value, the reduction in travel time for defectors ($t_F < t^{SO}$) is at the cost of cooperators in terms of increased travel time. The increased travel time is compensated for by the re-distributed defector payment. It can be shown mathematically that defectors and cooperators have the same generalized cost (combining time and monetary costs) equal to $t^{SO}$ (in time units). Intuitively, the payment is a transfer within the system and thus does not affect total system cost, which remains the SO.
total travel time. The generalized costs of a defector and cooperator are equal as there are positive numbers of both (non-corner solution), and as a result, they must be equal to the average SO travel time. When \( n = x_F \), the cooperative scheme degenerates to traditional congestion pricing with toll re-distribution, where no turn-taking is happening as Fast Route is filled up by defectors. Note that a cooperative scheme that does not maintain an SO flow pattern (but still better than UE in total travel cost) is still possible when \( n = x_F \).

### Heterogeneous VOT

Realistically, VOT is heterogeneous among travelers. Let \( \beta \) be the random VOT distributed over travelers. If the support of \( \beta \) has an upper bound \( \hat{\beta} \), it is trivial to show that a cooperative scheme (maintaining SO flow pattern) always exists with the defector penalty \( \tau \geq \hat{\beta}(t_S - t_F)x_S/d \). This is a much milder condition compared to those for Pareto-improving congestion pricing with toll re-distribution [16]. An added advantage, as mentioned previously, is that no financial transactions are needed and thus the dooming perception of “tax” is avoided.

Practical considerations might lead to an upper bound on the defector penalty, for example, to avoid the perception of forced cooperation with the government. When \( \tau < \hat{\beta}(t_S - t_F)x_S/d \), a traveler with the threshold VOT, \( \bar{\beta} = (1/\tau)(t_S - t_F)x_S/d \), is indifferent between cooperation and defection. Travelers with a VOT higher than \( \bar{\beta} \) will defect while those with a VOT lower than \( \bar{\beta} \) will cooperate. The existence condition of the scheme is thus a condition to ensure that the number of defectors is no larger than \( x_F \). With the same re-distribution scheme, it can be shown that every traveler is better off compared to UE\(^1\), although the generalized cost is not equalized among travelers due to heterogeneous VOT.

A potentially more appealing scheme is to set the penalty to a value high enough but not too high, so that a certain number of defectors exist while the travel time of cooperators is strictly better than that in UE. It appeals to high-VOT travelers by giving them an option to pay for better travel time; it appeals to low-VOT travelers by reducing their travel time and on top of that, providing monetary rewards (re-distributed penalty). The overall scheme would thus be less likely viewed as authoritarian. The number of defectors to ensure a strictly improving travel time for cooperators is such that

\[
 n < \frac{x_F(t_S - t_F) - d(t_S - t^{UE})}{t^{UE} - t_F}.
\]

Under the condition that \( x_F(t_S - t_F) > d(t_S - t^{UE}) \), such an \( n \) always exists. Let \( F_\beta(\cdot) \) be the cumulative distribution function of \( \beta \). The penalty corresponding to a given number of defectors \( n \) is \( F_\beta^{-1}(\frac{d - n}{d})(t_S - t_F)x_S/d \).

### A MATHEMATICAL PROBLEM FORMULATION FOR OPTIMAL CS IN A GENERAL NETWORK

The naive CS in the previous section cannot be applied to a real network, as the cycle length must be short enough to gain public acceptance. Integer non-linear programming problem formulations are proposed to

\(^1\)The traveler with threshold VOT, \( \bar{\beta} \), has a generalized cost of \( t^{SO} \) that is strictly better than \( t^{UE} \). S/he can serve as a reference for a regular defectors or cooperators whose VOT is different from \( \bar{\beta} \). A regular defector has a higher VOT and the same travel time as the threshold defector. It follows that the travel time saving \( t^{UE} - t_F \) is more valuable than for the threshold defector in terms of offsetting the penalty. The threshold defector is better off and thus the regular defector is also better off. A regular cooperator has a lower VOT and the same travel time as the threshold cooperator. When the regular cooperator’s travel time is lower than that in UE, s/he is better off since the re-distributed penalty can only further reduce the generalized cost. When the regular cooperator’s travel time is higher than that in UE, the increase in travel time is less detrimental than for the threshold cooperator in terms of offsetting the re-distributed penalty. The threshold cooperator is better off and thus the regular cooperator is also better off.
find optimal CS in a general network with a given cycle length.

**Formulation 1**

Let $L$ be the set of links in the network, $I^{mn}$ the set of travelers (all assumed to be cooperators) between OD pair $(m,n)$ and $D$ the fixed cycle length. $t_a(.)$ is the volume-delay function on link $a$ as a function of the flow on link $a$. A binary variable, $x_{adi}^{mn}$ is defined to indicate whether a traveler is on a particular link on a given day.

$$x_{adi}^{mn} = \begin{cases} 
1, & \text{if individual 'i' of OD pair (m,n) is on link 'a' on day 'd'} \\
0, & \text{otherwise}
\end{cases}$$

The average daily total travel time of the system $Z(x)$, is calculated as

$$Z(x) = \frac{1}{D} \sum_d \sum_a t_a(\sum_{mn,i} x_{adi}^{mn}) \sum_{mn,i} x_{adi}^{mn} / D$$ (4)

We set up a set of equity constraints to ensure that the CS is equitable to the cooperators.

$$\left| \frac{1}{D} \sum_d \sum_a t_a(\sum_{mn,i} x_{adi}^{mn}) x_{adi}^{mn} - u_{mn} \right| \leq \epsilon u_{mn}, \forall (m,n), \forall i \in I^{mn}$$ (5)

where

$$u_{mn} = \frac{1}{D * |I^{mn}|} \left( \sum_d \sum_a t_a(\sum_{mn,i} x_{adi}^{mn}) \sum_{mn,i} x_{adi}^{mn} \right)$$ (6)

Equation 5 ensures that differences of average travel time among cooperators over a cycle falls within a certain threshold (e.g. 5%). $u_{mn}$ is the average travel time between OD $(m,n)$ over a cycle.

$$\sum_{a \in A(m)} x_{adi}^{mn} = 1, \forall (m,n), \forall i, \forall d$$ (7)

$$\sum_{a \in B(n)} x_{adi}^{mn} = 1, \forall (m,n), \forall i, \forall d$$ (8)

$$\sum_{a \in A(k)} x_{adi}^{mn} = \sum_{a \in B(k)} x_{adi}^{mn}, \forall k \neq m,n, \forall i, \forall d$$ (9)

Equations 7 through 9 are the flow conservation constraints. Equation 7 implies that the flow out of an origin (m) has to be exactly equal to 1 for any person (i) on any given day (d). Equation 8 implies that the flow incoming to a destination (n) has to be exactly equal to 1 for any person (i) on any given day (d). Equation 9 implies that the inflow is equal to outflow at any intermediate node for any person(i) on any given day(d).

Therefore, the cooperative scheme is formulated as a constrained non-linear optimization problem,

$$\text{P1} \quad \min_{x_{adi}^{mn}} Z(x)$$

s.t. \quad Eqs. (5) – (9)

$$x \in \{0, 1\}$$
The above formulation is an integer non-linear programming problem which is difficult to solve due to the combination of non-convexity of Eq. (5) and the integrality constraints. We implemented a Lagrangian relaxation algorithm to solve the problem and tested it in small problem instances using Matlab. However the efficiency and effectiveness of the algorithm is still a major concern and thus an alternative formation is proposed that replaces the equity constraints (Eq. 5) with Pareto-improving constraints.

\[
\frac{1}{D} \sum_d \sum_a t_a (\sum_{mn,i} x_{adi} x_{adi}^m) x_{adi}^m \leq t_{UE}, \forall (m,n), \forall i \in I^{mn}
\]  

(10)

where the the average travel time of cooperators over a cycle is on greater than the UE travel time for any OD.

Formulation 2 is as follows:

\[
P_2 \min_{x_{adi}} Z(x) \\
\text{s.t. } Eqs. (10), (7) - (9) \\
x \in \{0, 1\}
\]

**COMPUTATIONAL TESTS**

P2 is easier to solve in that constraint (10) is convex. However the number of integer decision variables, which is the product of the total demand (integer), cycle length and number of links, makes the problem still difficult. A similar strategy as in the theoretical analysis is adopted where the travelers are divided into groups with approximately the same size, and the binary decision variables are defined for groups instead of individuals.

We used BONMIN (Basic Open-source Nonlinear Mixed INteger programming) which is an open-source C++ code for solving general MINLP (Mixed Integer NonLinear Programming), run remotely through the NEOS web interface.

We tested P2 in a single-OD two-route network with the following volume-delay functions:

\[
t_1 = 2 + \left( \frac{x_1}{3000} \right)^2, t_2 = 12 + \frac{x_1}{3000}
\]

Sensitivity analysis is done by starting with a demand of 9000 users and then scaling it in the range of 0.5 to 1.5. Different group sizes ranging from 3 to 5 and cycle lengths in the range of 3 to 5 are also tested.

Table 1 shows the optimal turn taking strategy for 9000 users divided into three groups. It can be seen that three groups of cooperators achieve the SO flow pattern by taking turns within a cycle length of 3 days. With increased cycle length of 4 and 5 days respectively the resulting average travel times are still close to SO and Pareto-improving for each groups. The defector penalty is calculated by taking the difference between a free riders (defector) travel time and the highest average travel time of a cooperator. A constant VOT of 50$/hr is considered for calculating the defector penalty.

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Figure 3 shows a barplot of average travel time of 3 groups of cooperators at a demand level 9000. The red horizontal line in Figure 3 represents the UE travel time. It can be seen that each group of cooperators for this demand level has strictly better travel time than UE.

In order to evaluate the efficiency and equity standard of the cooperative scheme we used two matrices: % of maximum travel time improvement (% Max), defined as the difference in average travel time between UE and SO, and the GINI coefficient (0 indicates perfect equity, and 1 inequity). Table 2 shows that when demand is low (4500 and 6750), SO and UE are the same and there is no room for
<table>
<thead>
<tr>
<th>Day</th>
<th>Link 1</th>
<th>Link 2</th>
<th>Flow on link 1</th>
<th>Flow on link 2</th>
<th>Defector penalty, $</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group 1</td>
<td>Group 2</td>
<td>Group 3</td>
<td>Group 1</td>
<td>Group 2</td>
</tr>
<tr>
<td>Day 1</td>
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<td>1</td>
<td>0</td>
<td>1</td>
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<tr>
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<td>0</td>
<td>0</td>
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<td>1</td>
<td>1</td>
<td>0</td>
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<td>2</td>
<td>1</td>
<td>1</td>
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<tr>
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<td>0</td>
<td>0</td>
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</tr>
<tr>
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<td>0</td>
<td>1</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
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<td># of Days on Link</td>
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<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
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<tr>
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<tr>
<td># of Days on Link</td>
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<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
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</table>
FIGURE 3  Average travel time of 3 groups of cooperators over different cycle length
improvement. When the demand is high enough to create efficiency difference between SO and UE, it seems that the higher the demand, the smaller the possible gain from turn taking. GINI indices in all cases are fairly small and achieves 0 with 3 groups, indicating that the equity issue is well accounted for. Table 3 shows that a larger number of groups generally improves both the efficiency and equity of the CS, which is reasonable given that a larger number of groups allows for more flexibility in the turn-taking.

**TABLE 2 Performance metrics at different demand level and cycle lengths (3 groups)**

<table>
<thead>
<tr>
<th>Cycle length (day)</th>
<th>Demand = 4500</th>
<th>Demand = 6750</th>
<th>Demand = 9000</th>
<th>Demand = 11250</th>
<th>Demand = 13500</th>
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<tbody>
<tr>
<td>% Max</td>
<td>GINI</td>
<td>% Max</td>
<td>GINI</td>
<td>% Max</td>
<td>GINI</td>
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<td>3</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>89</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>81</td>
<td>0.022</td>
<td>89</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>85</td>
<td>0.046</td>
<td>89</td>
</tr>
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</table>

**TABLE 3 Performance metrics with different group sizes and cycle lengths (demand = 9000)**

<table>
<thead>
<tr>
<th>Cycle length (day)</th>
<th>Number of Groups = 3</th>
<th>Number of Groups = 4</th>
<th>Number of Groups = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Max</td>
<td>GINI</td>
<td>% Max</td>
<td>GINI</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
<td>0.021</td>
<td>96</td>
</tr>
<tr>
<td>5</td>
<td>85</td>
<td>0.046</td>
<td>96</td>
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</table>

**CONCLUSIONS AND FUTURE DIRECTIONS**

This research extends the rationing and pricing scheme to a general network to cope with the equity issue when driving a transportation system from UE to SO. In the theoretical analysis, it is shown that when the value of time (VOT) is bounded from above, a Pareto-improving cooperative scheme without financial transactions always exists, in which case the defector penalty is high enough so that all travelers cooperate. A more practical and potentially more appealing case is discussed where a certain number of defectors exist while the travel time of cooperators is strictly better than that in UE. A mathematical programming problem is formulated for the optimal cooperative scheme problem in a general network with Pareto-improving constraints and practical considerations on the length the cooperation cycle. Computational tests on a simple network and solutions are evaluated in terms of efficiency improvement (total system travel time) and equitability (Gini index).

Future research directions include 1) systematic testing of the CS as formulated (P2) in realistic networks, 2) extension of the analysis to probabilistic route choice so that stochastic UE (SUE) emerges without interventions, and 3) extension of the formulation to allow a certain fraction of defectors so that the system will not be viewed as too authoritarian.

**REFERENCES**


